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INTERESTING GEOMETRY

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Abstract
Through this article, we will provide readers with important information about geometric problems and their solutions.

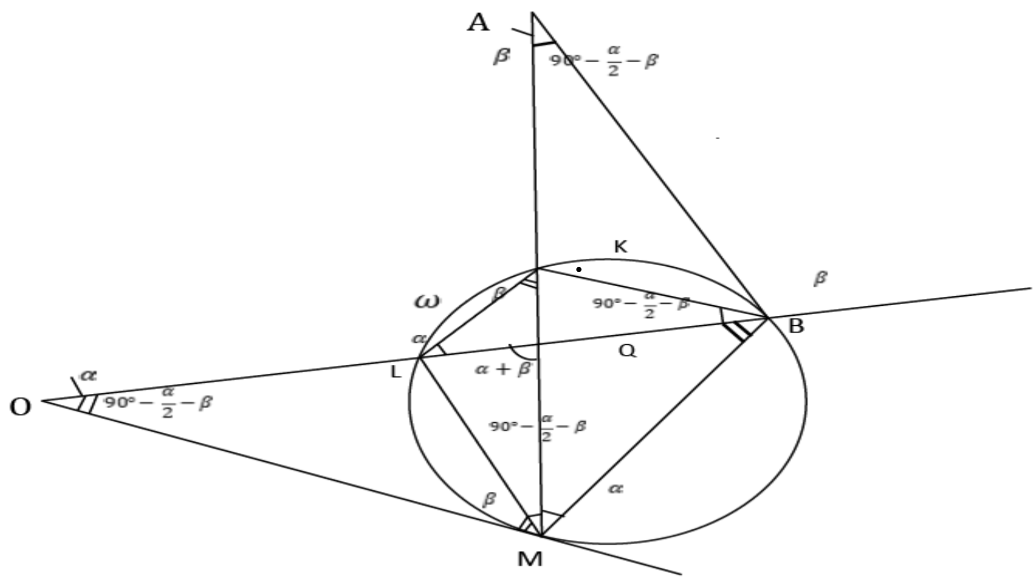
Keywords: circle, angle, bisector, triangle.

Task 1.

Circle G is inscribed inside the angle with its tip at point O and touches the sides of the angle at points A and B. Point K is an arbitrary point on the $\breve{AB}$ (small) arc. On the straight line OB, there is a point L such obtained, that here $OA \parallel KL$ .The circle drawn outside of ΔKLB - ω .The straight line AK intersects ω for the second time at the point M.

Prove it; The straight line MO is tangent to ω circle.

Proof.



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According to the condition:

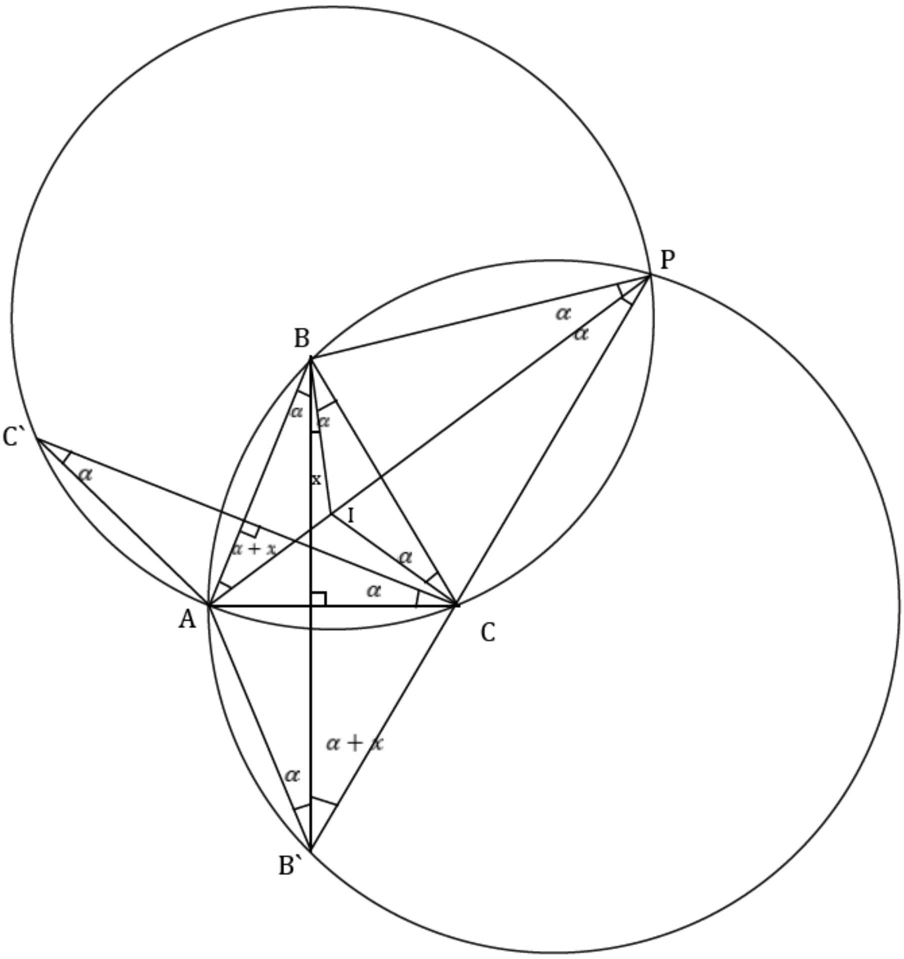
$OA \parallel KL \Rightarrow \angle AOL = \alpha \; va \; \angle OAK = \beta \Rightarrow \angle KLB = \alpha \; \angle LKM = \beta. \; OA = OB \Rightarrow \angle OAB = 90^\circ - \frac{\alpha}{2} \Rightarrow \angle KAB = 90^\circ - \frac{\alpha}{2} - \beta \Rightarrow \angle KBO = \angle KML = 90^\circ - \frac{\alpha}{2} - \beta. \angle KLB = \angle KMB = \alpha \; va \; \angle LKM = \angle LBM = \beta \Rightarrow OABM - cyclyc. \Rightarrow \angle MAB = \angle MOB = 90^\circ - \frac{\alpha}{2} - \beta. \angle LQM = \alpha + \beta \; (LB \cap KM = Q). \Rightarrow \angle OMQ = \beta \Rightarrow$ *the straight line MO is tangent to cycle*

ω $. \Rightarrow$ ▲

Task 2.

$\Delta ABC -$acute angled. Points B\` and C\` are symmetrical points of points B and C with respect to straight lines AC and AB, respectively. The circles drawn outside OF $\Delta ABB\grave{} \; va \; \Delta ACC$ intersect for the second time at the point P. Prove;

The center of the circle drawn outward to ΔABC lies on the straight line PA.



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Proof;

$CC` \cap BB` = M . \Rightarrow \angle ABB` = \angle ACC` = \angle AB`B = \angle AC`C = \alpha \Rightarrow \angle AC`C = \angle APC = \alpha$ and $\angle ABB` = \angle APB` = \alpha \Rightarrow B`, C$ and P points are collinear in PA $AI = BI$ so that we get point $I. \angle IBB` = x$. $\Rightarrow \angle ABI = \angle BAI = \alpha + x \Rightarrow \angle BAP = \angle BB`P = \alpha + x. CB` = BC \Rightarrow \angle B`BC = \alpha + x \Rightarrow \angle IBC = \alpha$ va $\angle IPC = \alpha \Rightarrow BICP - cyclyc$ $\angle AB`B = \angle APB = \alpha \Rightarrow \angle BPI = \angle BCI = \alpha \Rightarrow BI = IC \Rightarrow AI = BI = IC \Rightarrow$ Point I is center of the circle drawn outward to ΔABC and $I \in PA \Rightarrow$ ▲

Task3.

In triangle ΔABC $AB = AC$. The bisectors of the angles $\angle CAB$ and $\angle ABC$ intersect the sides at points D and E, respectively. Point K be the center of the ΔADC inscribed circle. If $\angle BEK = 45°$, find all possible values of the angle $\angle CAB$.

Solution;

$AD \perp BC.$ $BE \cap KC = M.$ $M \in AD. \angle MOK = 45°. \angle MCD = 45° - \frac{\alpha}{2}. \Rightarrow \angle DMC = 45° + \frac{\alpha}{2}, \Rightarrow \angle EMC = 90° - \alpha. \angle AME = 45 + \frac{\alpha}{2} \Rightarrow \angle KEC = \frac{3\alpha}{2}$

We use the theorem of sines.(MK=c DK=a KE=b KC=d)

$$\frac{a}{\cos(45° - \alpha)} = \frac{c}{\sin 45°} = \frac{b}{\cos\alpha} \Rightarrow \frac{a}{b} = \frac{\cos(45° - \alpha)}{\cos\alpha} \qquad (1)$$

$$\frac{a}{\sin(45° - \alpha)} = \frac{d}{\sin 45°}, \quad \frac{b}{\sin(45° - \alpha)} = \frac{d}{\sin\frac{3\alpha}{2}} \quad \Rightarrow \quad \frac{a}{b} = \frac{\sin\frac{3\alpha}{2}}{\sin 45°} = \sqrt{2}\cdot\sin\frac{3\alpha}{2} \qquad ()$$

$$(1) va (2) \Rightarrow \cos(45° - \alpha) = \sqrt{2}\cdot\cos\alpha\cdot\sin\frac{3\alpha}{2} \Rightarrow \cos\frac{\alpha}{2} + \sin\frac{\alpha}{2} = 2\cdot\cos\alpha\cdot\sin\frac{3\alpha}{2}$$
$$= \sin\frac{5\alpha}{2} + \sin\frac{\alpha}{2} \Leftrightarrow \cos\frac{\alpha}{2} = \sin\frac{5\alpha}{2} \Leftrightarrow \sin\frac{5\alpha}{2} = \sin\left(90° - \frac{\alpha}{2}\right) \Rightarrow \frac{5\alpha}{2}$$
$$= 90° - \frac{\alpha}{2} \ yoki\ \frac{5\alpha}{2} + 90° - \frac{\alpha}{2} = \pi.$$

1) $\frac{5\alpha}{2} = 90° - \frac{\alpha}{2} \quad \Rightarrow \quad \alpha = 30° \ \Rightarrow \angle CAB = 60°$

2) $\frac{4\alpha}{2} = 90° \Rightarrow \quad 2\alpha = \angle CAB = 90°$

Answer;$\angle CAB = 60°$ or $\angle CAB = 90°$

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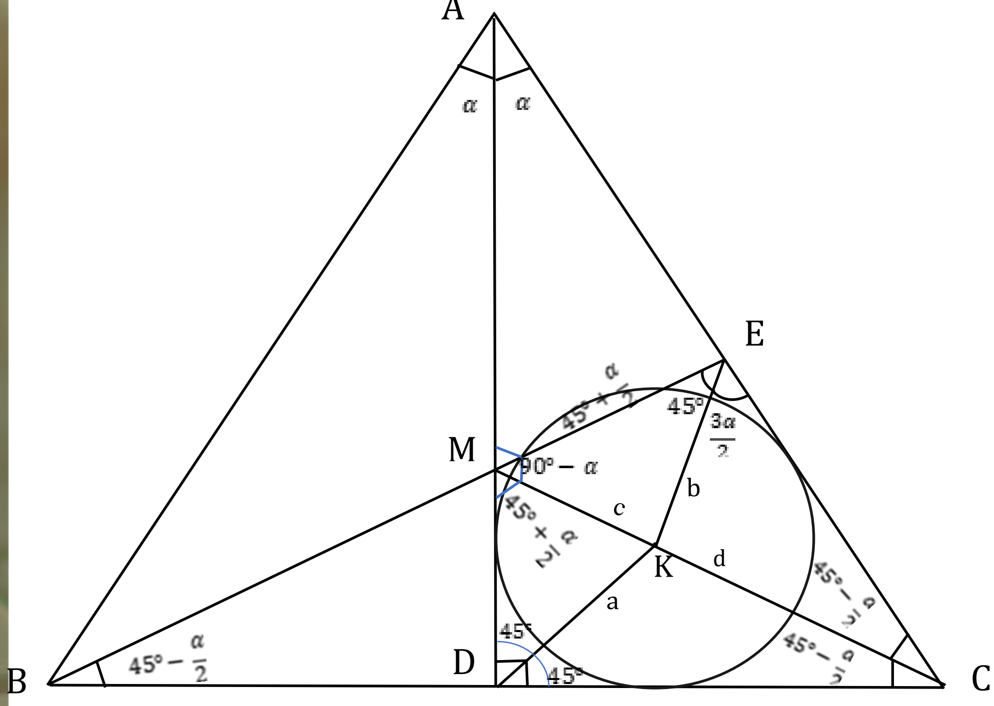
Proof;

$CC` \cap BB` = M. \Rightarrow \angle ABB` = \angle ACC` = \angle AB`B = \angle AC`C = \alpha \Rightarrow \angle AC`C = \angle APC = \alpha$ and $\angle ABB` = \angle APB` = \alpha \Rightarrow B`, C$ and P points are collinear PA da $AI = BI$

so we get point I $\angle IBB` = x$.

$\Rightarrow \angle ABI = \angle BAI = \alpha + x \Rightarrow \angle BAP = \angle BB`P = \alpha + x. CB` = BC \Rightarrow \angle B`BC = \alpha + x \Rightarrow \angle IBC = \alpha$ va $\angle IPC = \alpha \Rightarrow BICP - cyclyc \angle AB`B = \angle APB = \alpha \Rightarrow \angle BPI = \angle BCI = \alpha \Rightarrow BI = IC \Rightarrow AI = BI = IC \Rightarrow$ Point I is

center of the circle drawn outward to ΔABC and $I \in PA \Rightarrow$ ▲



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