INNOVATIVE TECHNOLOGICA
METHODICAL RESEARCH JOURNAL
ISSN: 2776-0987 Volume 3, Issue 2, Feb., 2022

A GENERALIZATION OF THE GAUSSIAN PRINCIPLE OF LEAST COERCION FOR SYSTEMS WITH NON-IDEAL CONNECTIONS

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Abstract

The issue of extending the extended constraint combination method to nonholonomic systems with nonideal constraints is considered. A generalization of the Gauss principle of least constraint for systems with non-ideal constraints (for systems with friction) is given in the case when possible displacements satisfy the conditions of the extended method of combining constraints.

Keywords: non-ideal bonds; combination of connections; friction forces; strength of connections; possible movements; stability, friction adjuster, stationary motion

INTRODUCTION

Consider a mechanical system of N material points M_K $(k=1,2,...,N)$ m_k with masses , whose position relative to the inertial Cartesian coordinate system is determined by the radius-vectors $\vec{r}_k(x_\gamma)$ $(\gamma=1,2,...,3N)$. The system is under the action $\vec{F}_k(X_\gamma)$ of given forces and is constrained by joint and independent connections, among which there are both geometric

$$f_\alpha(x_1, x_2, ..., x_{3N}, t) = 0 \quad (\alpha = 1,...,a), \tag{1}$$

and kinematic, generally speaking, non-linear

$$\varphi_\beta(x_\gamma, \dot{x}_\gamma, t) = 0 \quad (\beta = 1,...,b). \tag{2}$$

The possible displacements allowed by the constraints are determined by the independent relations

$$\sum_{\gamma=1}^{3N} \frac{\partial f_\alpha}{\partial x_\gamma} \delta x_\gamma = 0, \quad \sum_{\gamma=1}^{3N} \frac{\partial \varphi_\beta}{\partial \dot{x}_\gamma} \delta \dot{x}_\gamma = 0 \qquad (3)$$

Let us assume that the sum of the elementary work of the reaction forces on any possible displacement is nonzero

$$\sum_{k=1}^{N} \vec{R}_k \delta \vec{r}_k \neq 0. \qquad (4)$$

It is known that the bonds in this case will refer to the bonds with friction.

We decompose $\vec{R}_k$ the bond reaction acting on the point M_k into two components of the force $\vec{R}_k^\tau$ and $\vec{R}_k^n$, which have the following properties: $\delta \vec{r}_k$

1. On every possible movement of the system

$$\sum_{k=1}^{N} \vec{R}^n{}_k \delta \vec{r}_k = 0, \quad \sum_{k=1}^{N} \vec{R}_k^\tau \delta \vec{r}_k \neq 0. \qquad (5)$$

Informing the system of an arbitrary possible displacement, by virtue of condition (5), we obtain the equation:

$$\sum_{k=1}^{N} (\vec{F}_k + \vec{R}_k^\tau - m_k \vec{w}_k) \delta \vec{r}_k = 0. \qquad (6)$$

which expresses the general equation of dynamics for the considered systems with nonideal constraints.

As applied to systems with friction, equation (6) is a necessary and sufficient condition for the correspondence to the given forces of the system's motion compatible with the constraints, given the known system friction law. To do this, differentiating equations (1) two times with respect to time, and equations (2) once, we obtain:

$$\sum_{\gamma=1}^{3n} \frac{\partial f_\alpha}{\partial x_\gamma} \ddot{x}_\gamma + A_\alpha = 0 \qquad (\alpha = 1, \ldots, a), \qquad (7)$$

$$, \sum_{\gamma=1}^{3n} \frac{\partial \varphi_\beta}{\partial \dot{x}_\gamma} \ddot{x}_\gamma + B_\beta = 0 \qquad (\beta = 1, \ldots, b) \qquad (8)$$

where , terms that do not contain accelerations, and are the actual $\ddot{x}_\gamma$ -accelerations of the points of the system.

We denote by $\ddot{x}_\gamma$ kinematically possible accelerations, that is, accelerations compatible with constraints (1) and (2). The latter will satisfy the conditions:

$$\sum_{\gamma=1}^{3n} \frac{\partial f_\alpha}{\partial x_\gamma} \ddot{x}'_\gamma + A_\alpha = 0 \quad (\alpha = 1, \ldots, a), \qquad (9)$$

$$\sum_{\gamma=1}^{3n} \frac{\partial \varphi_\beta}{\partial \dot{x}_\gamma} \ddot{x}'_\gamma + B_\beta = 0 \ (\beta = 1,\dots,b). \quad (10)$$

Since they are A_α *and* B_β functions of time, coordinates and velocities, then from (6); (9), (10) we get

$$\delta \ddot{x}_\gamma = \ddot{x}_\gamma - \ddot{x}'_\gamma ,$$

$$\sum_{\gamma=1}^{3n} \frac{\partial f_\alpha}{\partial x_\gamma} \delta \ddot{x}_\gamma = 0 ,$$

$$\sum_{\gamma=1}^{3n} \frac{\partial \varphi_\beta}{\partial \dot{x}_\gamma} \delta \ddot{x}_\gamma = 0 . \quad (11)$$

Comparing these expressions with the conditions for possible displacements (3), we see that the acceleration variations satisfy the same conditions as the possible displacements. Therefore, from (6) we get:

$$\sum_{\gamma=1}^{3n} \left(m_\gamma \ddot{x}_\gamma - X_\gamma - R_\gamma^\tau \right) \left(\ddot{x}_\gamma - \ddot{x}'_\gamma \right) = 0 . \quad (12)$$

The movement of the system, which it will perform under the action of given forces $\overline{F}_k$ and forces equal to the forces of friction $\vec{R}_k^\tau$, will be called the actual released movement. We denote the accelerations of points in the actual freed motion by a_γ $(\gamma = 1,2,\dots,3N)$.

Since the general equation of dynamics is also valid for a liberated system, the expression takes place:

$$\sum_{\gamma=1}^{3n} \left(m_\gamma a_\gamma - X_\gamma - R_\gamma^\tau \right) \left(\ddot{x}_\gamma - \ddot{x}'_\gamma \right) = 0 . \quad (13)$$

Now subtracting (12) from (13), we obtain

$$\sum_{\gamma=1}^{3n} m_\gamma \left[\left(\ddot{x}_\gamma - a_\gamma \right) \left(\ddot{x}_\gamma - \ddot{x}'_\gamma \right) \right] = 0. \quad (14)$$

The last relation can be converted to the form:

$$\sum m_\gamma \left[\left(\ddot{x}_\gamma - a_\gamma \right) \left(\ddot{x}_\gamma - \ddot{x}'_\gamma \right) \right] = \sum \frac{m_\gamma}{2} \begin{bmatrix} \left(\ddot{x}_\gamma^2 - 2\ddot{x}_\gamma \ddot{x}'_\gamma + x_\gamma'^2 \right) - \left(\ddot{x}_\gamma'^2 - 2\ddot{x}'_\gamma a_\gamma + a_\gamma^2 \right) + \\ + \left(a_\gamma^2 - 2\ddot{x}_\gamma a_\gamma + \ddot{x}_\gamma^2 \right) \end{bmatrix}. \quad (15)$$

If we now introduce deviation measures [6] (definition according to Gauss)

$$A_{dD} = \sum \frac{m_\gamma}{2} (\ddot{x}_\gamma - a_\gamma)^2 ,$$

ISSN: 2776-0987 Volume 3, Issue 2, Feb., 2022

$$A_{\mu g} = \sum \frac{m_\gamma}{2}(a_\gamma - \ddot{x}'_\gamma)^2 ,$$

$$A_{d\mu} = \sum \frac{m_\gamma}{2}(\ddot{x}_\gamma - \ddot{x}'_\gamma)^2 ,$$

then from (10) we get

$$A_{d\mu} + A_{dD} - A_{\mu g} = 0 \quad . \tag{16}$$

Since each term of the last relation is non-negative, the conditions must be satisfied

$$A_{d\mu} \le A_{\mu g}, \; A_{dD} \le A_{\mu g} . \tag{17}$$

The second of these inequalities is a generalization of the Gauss principle of least constraint for systems with non-ideal constraints. According to this principle, among the possible accelerations, the real accelerations of the points of the system with non-ideal constraints minimize the function

$$A_{d\mu} = \frac{1}{2}\sum_{\gamma=1}^{3n} m_\gamma \left(\ddot{x}_\gamma - \frac{X_\gamma + R_\gamma^\tau}{m_\gamma} \right)^2 . \tag{18}$$

Thus, according to the obtained Gauss principle, among all conceivable accelerations, the real accelerations of the points of systems with friction turn the function (18) to a minimum and vice versa, the conditions for the minimum of the function (18) in terms of accelerations that satisfy conditions (9) and (10) lead to differential equations of the actual motion of a system with non-ideal constraints

$$\ddot{x}_\gamma = \frac{1}{m_\gamma}\left(F_\gamma + \sum_{\alpha=1}^{a} \lambda_\alpha \frac{\partial f_\alpha}{\partial x_\gamma} + \sum_{\beta=1}^{b} u_\beta \frac{\partial \varphi_\beta}{\partial \dot{x}_\gamma} + \sum_{j=1}^{r} \mu_j \frac{\partial (m_\gamma \dot{x}_\gamma)}{\partial p_j} \right) . \tag{19}$$

Thus, the extended constraint combination method is extended to nonholonomic systems with nonideal constraints. It is shown that for such systems there is a general equation of dynamics, which allows us to generalize the Gauss principle of least constraint.

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